# Mechanical Properties of Twisted Filaments

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## Synopsis

The knowledge of some mechanical properties of twisted filament is necessary to predict properties of continuous-filament yarns. Retraction, strength, strain, and elastic properties of filaments are the chief characteristics. Assuming that the filament can be considered as a yarn made of fibers bounded together and then twisted, formulas for prediction of those properties were elaborated and then compared with the experimentally received values. Satisfactory agreement of computed results with experimental values is obtained if the changes in volume of filament at strain is taken into account.

## INTRODUCTION

The progress in textile technology and the extension of the applications of textile raw materials and products necessitate comprehensive analysis of the phenomena associated with the production, processing, and use of textile fibers, yarns, and products. Ever since the chemical fibers were introduced on the market, the proportion of filament yarns in the total quantum of textile raw materials has been steadily growing. The filament yarns often must be twisted, and in this operation the elementary fibers are both twisted and bent. The present article is an attempt to throw some light upon the retraction and changes in the tensile properties of twisted monofilaments and changes in the elastic properties of bifilaments obtained by doubling and twisting two twistless monofilaments.

## RETRACTION OF FILAMENTS IN TWISTING

A filament can be viewed as a body composed of fibrils that are held together by forces of cohesion. Initially, the axes of the fibrils are parallel with the filament axis. In this state, filament is similar to a yarn produced by adhesive bonding of the fibers, as evidenced by photographs of axially shredded filaments, Figure 1(a). In twisting, the fibers adopt a helical form. Evidence of this is again found in photographs of axially shredded filaments which had been previously twisted and stretched till rupture, Figure 1(b). Thus, it follows that in every layer of the fibrils of a twisted filament there must be present longitudinal and shear strains the values of which are proportional to the distances of the fibrils from the filament axis.

Let us imagine that from a twisted filament of circular cross section a segment has been cut of a length equal to the pitch of the spiral formed by the fibrils on the filament surface, assuming that originally the fibrils were positioned parallel with the filament axis. Let the angle formed between a tangent to this spiral and the direction of the filament axis (Fig. 2) be designated as $\beta_R$. The obvious relation:

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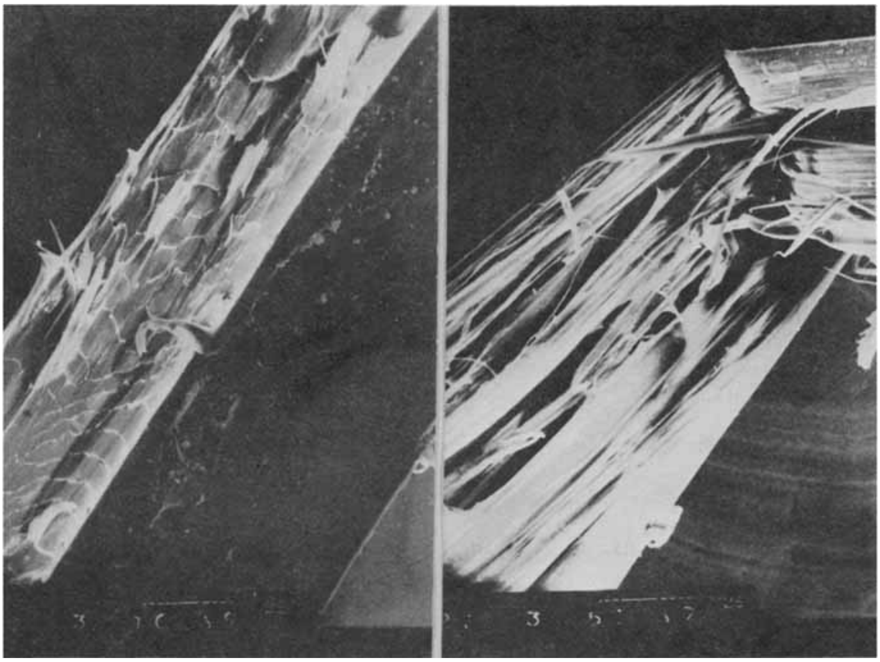
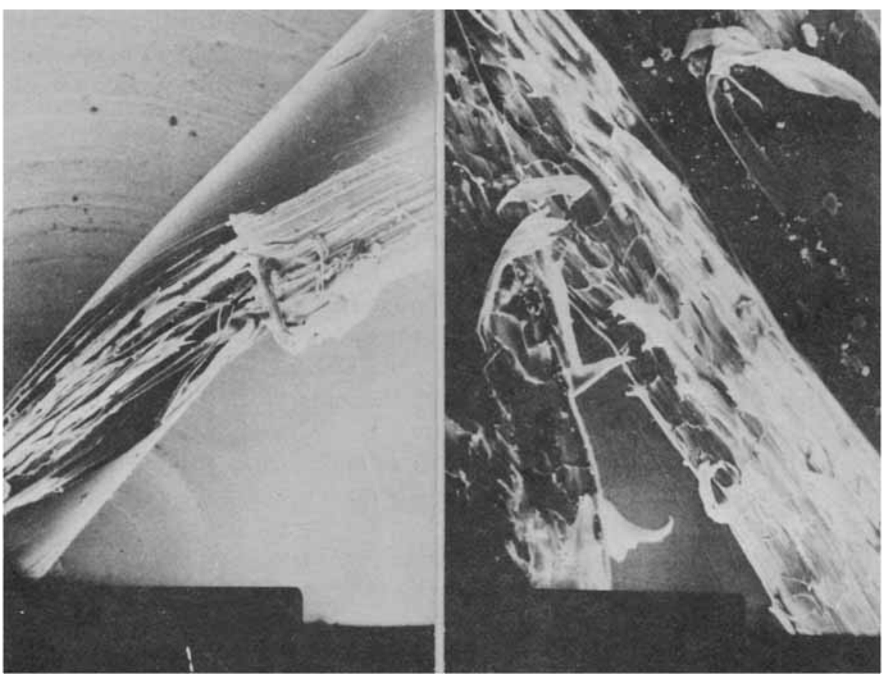


(a)



(b)

Fig. 1. (a) Axially shredded twistless filament (the fibrils are parallel to filament axis). (b) Twisted filament, axially shredded upon stretching to rupture (the shredding is spiral; fibrils have a spiral configuration).

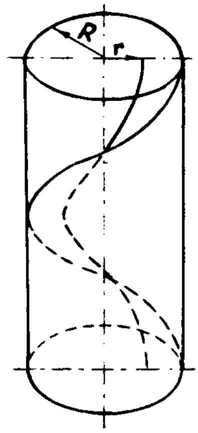



Fig. 2. Schematic representation of a twisted filament.

$$\text{tg}\,\beta_R = \frac{2\pi R}{h} = 2\pi RT \tag{1}$$

holds true then, where $R$ is the external radius of filament cross section, $h$ is the pitch of the spiral on filament surface, and $T$ is the number of turns per unit length of filament.

Because of the cohesion of the fibrils, the spiral pitch of the axes of the fibrils which are inside the filament body is equal to that of the axes of the surface fibrils. Therefore, for the fibrils that are a distance $L$ away from the filament axis,

$$\text{tg}\,\beta = \frac{2\pi r}{h} = 2\pi rT \tag{2}$$

If the expressions $\text{tg}\,\beta_R = g$ (which will be referred to as "twist parameter") and $r/R = x$ are introduced, then

$$\text{tg}\,\beta = xg \tag{3}$$

The length $l$ of a spiral of radius $r$ will be of course

$$l = \frac{h}{\cos\beta} = h\sqrt{1 + \text{tg}^2\,\beta} = h\sqrt{1 + x^2g^2} \tag{4}$$

and is, therefore, a function of the distance $x$ from the filament axis and of the filament twist parameter $g$.

In twisting, the diversification of the lengths of the filament-composing elements results in longitudinal strain of the elements and, therefore, in axial stresses. Were the filament length invariable, the load applied to it in twisting would have to be continuously increased, as was observed by Dent and Herle.[1] If, however, only a minimum load is applied, just enough to keep the filament straight throughout the twisting operation, then a shortening of the filament length is observed. This shortening is a result of the diverse stresses that are generated in the external (where the filament elements are stretched) and internal (where the filament elements are compressed) layers of the filament.

Let $s$ denote the factor of filament retraction after twisting:

$$s = \frac{L_1}{L_0} \tag{5}$$

where $L_0$ is the initial length of filament and $L_1$ is the filament length after twisting. A segment of twisted filament of a length equal to one pitch $h$ will have resulted from a segment of twistless filament of length $l_0 = h/s$.

The length of a fibril at a distance $r$ from the twisted filament axis is

$$l = \frac{h}{\cos \beta} \quad (6)$$

The longitudinal strain of the fibril is

$$\epsilon = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 = \frac{h}{\cos \beta} \cdot \frac{s}{h} - 1 = s \sqrt{1 + x^2 g^2} - 1 \quad (7)$$

With small axial strains it is possible to adopt a directly proportional relation between stress and strain, the same for stretching and compression. Let $E$ be factor of proportionality – Young's modulus. Then, the relation between the axial stress and strain will be

$$\sigma = E\epsilon = E\,(s \sqrt{1 + x^2 g^2} - 1) \quad (8)$$

The stress is oriented along the fibril axis spiral, and it is calculated as stress per unit area of the fibril in a section normal to the fibril axis.

If $A$ denotes the fibril cross section area in a plane normal to a tangent to the fibril axis, the elementary force in the fibril, generated by stress $\sigma$, is

$$F' = \sigma A' \quad (9)$$

Its projection on the filament axis is

$$F'_a = \sigma A' \cos \beta \quad (10)$$

Because the fibril axis is at angle $\beta$ to the filament axis, the fibril cross-sectional area, in a plane normal to the filament axis, is

$$A'_a = \frac{A'}{\cos \beta} \quad (11)$$

Therefore, the component axial stress in the fibril along the filament axis is

$$\sigma_a = \frac{F'_a}{A'_a} = \frac{\sigma A' \cos^2 \beta}{A'} = \sigma \cos^2 \beta = E\,(s \sqrt{1 + x^2 g^2} - 1) \frac{1}{1 + x^2 g^2} \quad (12)$$

Such stresses are present in the fibrils which are a distance from $r = xR$ from the filament axis in a layer the thickness of which is $dr = R\,dx$. The total force generated in the elements is

$$dP = \sigma_a \cdot 2\pi r\,dr = E(s \sqrt{1 + x^2 g^2} - 1) \frac{1}{1 + x^2 g^2}$$
$$\cdot 2\pi x R^2\,dx = 2\pi R^2 \cdot E\,(s \sqrt{1 + x^2 g^2} - 1) \frac{1}{1 + x^2 g^2} \cdot x dx \quad (13)$$

The force resultant from the stresses that are present in the whole cross section of the yarn is

$$P = \int_0^1 2\pi R^2 E\,(s \sqrt{1 + x^2 g^2} - 1) \frac{1}{1 + x^2 g^2} x\,dx \quad (14)$$

and is equal to the filament weighting load (straightening force). From eq. (15) the required weighting load at predetermined retraction factor $s$ can be calculated:

$$P = \frac{E}{4\pi T^2} [2s (\sqrt{1+g^2} - 1) - \ln (1 + g^2)] \tag{15}$$

Especially if the stress–strain relationship is linear and the filament length in twisting is maintained constant ($s = 1$), the force necessary to preserve the condition is

$$P = \frac{E\pi R^2}{g^2} [2 (\sqrt{1+g^2} - 1) - \ln (1 + g^2)] \tag{16}$$

This permits anticipation of the increase of force $P$ as a function of twist parameter $g$. On the other hand, if a constant weighting load were used, the retraction factor would be

$$s = \frac{(\sigma_f/E)\, g^2 + \ln (1 + g^2)}{2(\sqrt{1+g^2} - 1)} = \frac{\epsilon g^2 + \ln (1 + g^2)}{2(\sqrt{1+g^2} - 1)} \tag{17}$$

where $\sigma_f = P/\pi R^2$ is filament tension and $\epsilon$ is filament strain at load $P$ (or at tension $\sigma_f$).

## STRENGTH OF TWISTED FILAMENTS

Obviously, if a filament is stretched, stresses are generated in it. In a twistless filament, the stress–strain interrelationship is generally nonlinear.

In adopting the earlier presented model of internal fibrilar structure of filaments, let us investigate what changes occur in an elementary cylinder having a diameter $2r$ and initial length $h$, in a twisted filament after the filament has been stretched to a length $h_1$.

As a result of stretching, the filament undergoes constriction (i.e., reduction of diameter). Its initial diameter $2R$ changes into diameter $2R_1$ (Fig. 3). It is

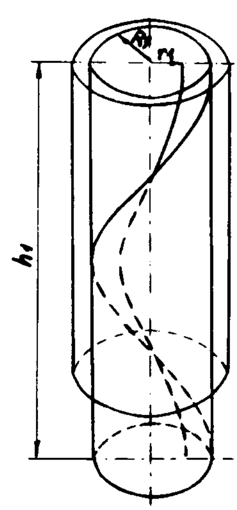



Fig. 3. Schematic representation of a twisted and stretched filament.

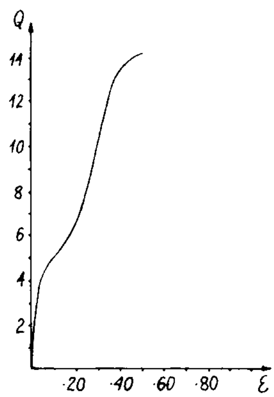


Fig. 4. Stress–strain relation for a filament.

reasonable to infer that the diameters of all the isolated layers will change similarly. Therefore,

$$\frac{2R_1}{2R} = \frac{2r_1}{2r} = u \tag{18}$$

where u is the filament constriction factor.

The spiral length in the stretched filament is

$$l_1 = \frac{h_1}{\cos \beta_1} \tag{19}$$

where $\beta_1$ is the angle between the fibril axis and the filament axis following stretching (prior to stretching, this angle was equal to $\beta$).

The value of the twist parameter is now

$$\text{tg}\, \beta_1 = \frac{2\pi R_1}{h_1} \tag{20}$$

If

$$\frac{h_1}{h} = 1 + a \tag{21}$$

where a is the strain of filament, then

$$\text{tg}\, \beta_{R1} = \frac{2\pi u R}{(1+a)\, h} = \frac{u}{1+a} g = \frac{g}{k} = g_1 \tag{22}$$

where $k$ is the coefficient of change of the geometry of filament fibrils. For a fibril the distance of which from the filament axis is $r = xR$, the twist parameter is

$$\text{tg}\, \beta_1 = x \frac{g}{k} = x g_1 \tag{23}$$

The strains which are produced in the filament result in the following longitudinal stress in the fibril:

$$\epsilon(x) = \frac{l_1 - l_0}{l_0} = \frac{h_1 s}{(\cos \beta_1)\, h} - 1 = s(1+a)\sqrt{1 + \text{tg}^2\, \beta_1} - 1 = s\sqrt{(1+a)^2 + u^2 x^2 g^2} - 1 \tag{24}$$

The strains are associated with stresses via relation $\sigma(\epsilon)$, which can be determined for twistless filament.

The total force in the layer originally a distance $l$ from the filament axis and calculated along the fibril axis is

$$dF = \sigma(\epsilon)\, 2\pi r\, dr \cos\beta \tag{25}$$

since the tension relates to the initial dimension of the body. A projection of the force on the stretched filament axis is

$$dP = dF \cos\beta_1 = \sigma(\epsilon)\, 2\pi r\, dr \cos\beta \cos\beta_1 \tag{26}$$

The total force to which the filament is subjected at strain $a$ is

$$P = \int_0^R \sigma(\epsilon)\, 2\pi\, r\, dr \cos\beta \cos\beta_1 = \pi R^2 \int_0^1 \sigma(\epsilon)\, 2x\, dx \cos\beta \cos\beta_1 \tag{27}$$

and the tension, as calculated in relation to the initial cross-sectional areas of the filament, is

$$\sigma_f = \frac{P}{\pi R^2} = \int_{x=0}^1 \sigma(\epsilon)\, 2\, x\, dx \cos\beta \cos\beta_1 \tag{28}$$

In order to find $\sigma_f$, it is necessary to know the function $\sigma(\epsilon)$. This function has a form that generally cannot be described by means of simple algebraic relations. However, for the particular values of the filament strain $a$, it is possible to determine the boundary values of fibril strain inside the filament [$\epsilon(0)$] and on the filament surface [$\epsilon(1)$] from the equations

$$\epsilon(0) = s(1 + a) - 1$$
$$\epsilon(1) = s\sqrt{(1 + a)^2 + u^2 g^2} - 1 \tag{29}$$

and in these intervals to approximate the tension by means of the parabolic function

$$\sigma = G + H\epsilon + J\epsilon^2 \tag{30}$$

If the above expression is substituted in eq. (30) in place of $\epsilon$ and in the right-hand side of eq. (24) and the thus expressed tension $\sigma$ is introduced into eq (28), then, upon integration, the filament tension $\sigma_f$ is obtained as a function of twist parameter $g$ and of filament strain $a$ (through parameter $k$) in the form:

$$\sigma_s = \frac{2k}{g^2}\left(\ln \frac{\sqrt{1+g^2} + \sqrt{k^2+g^2}}{1+k}\right)(G - H + J) + \frac{k}{g^2}[\ln(1+g^2)]u(H - 2J)$$
$$+ \frac{k}{g^2}\left(\frac{\ln\sqrt{1+g^2}}{\sqrt{1+g^2} - 1}\right)^2\left[\sqrt{(1+g^2)(k^2+g^2)} - k\right.$$
$$\left. + (k^2 - 1)\ln \frac{\sqrt{1+g^2} + \sqrt{k^2+g^2}}{1+k}\right]u^2 J \tag{31}$$

## STRAIN OF TWISTED FILAMENTS

Consistently with what has been already said, at filament strain $a$ the strain of filament fibrils, at a distance $r = xR$ from the filament axis, is

$$\epsilon = s\sqrt{(1+a)^2 + u^2x^2g^2} - 1 \tag{32}$$

The highest values of the strains are observed on the filament surface ($x = 1$). Therefore

$$\epsilon_{\max} = s\sqrt{(1+a)^2 + u^2g^2} - 1 \tag{33}$$

If it is assumed that the critical value of fibril strain $\epsilon_r$ is independent of the filament twist, then

$$(1 + a_r)^2 = \left(\frac{1+\epsilon_r}{s}\right)^2 - u^2g^2 \tag{34}$$

Hence, the twisted filament strain is

$$a_r = \sqrt{\left(\frac{1+\epsilon_r}{s}\right)^2 - u^2g^2} - 1 \tag{35}$$

## RESIDUAL STRAINS OF A TWISTED BIFILAMENT

It is best to characterize the elasticity of a twisted bifilament by its residual strains $a_{bp}$ after first straining the filament by $a_{tb}$.

In investigating the relationships between $a_{pb}$ and $a_{tb}$, let us study a bifilament twisted up of two twistless monofilaments (Fig. 5).

Let $T_b$ denote the number of turns of the bifilament. Thus, the monofilament axis spiral pitch is $h_b = 1/T_b$. At the same time, the monofilament axis spiral radius is equal to the monofilament cross-sectional radius $R$.

A monofilament twisted into a bifilament will have a twist equal to the monofilament axis torsion divided by $2\pi$. If the twist is designated by the symbol $T_2$, the result will be

$$T_2 = \frac{K}{2\pi} = \frac{1}{2\pi R}\cos\beta_b \sin\beta_b \tag{36}$$

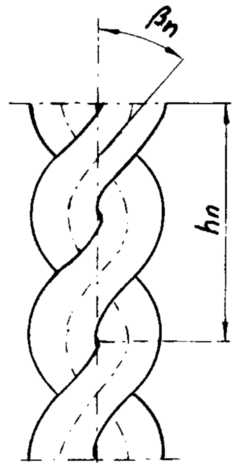



Fig. 5. Schematic representation of a twisted bifilament.

where $\beta_b$ is the angle formed between a tangent to the filament axis and the direction of the filament axis. If the bifilament twist parameter is written

$$g_b = 2\pi RT_b = \mathrm{tg}\,\beta_b \tag{37}$$

and the twist parameter of monofilament twisted into bifilament is

$$g_2 = 2\pi RT_2 \tag{38}$$

then

$$g_2 = \frac{g_b}{1 + g_b{}^2} \tag{39}$$

Let us investigate now a bifilament of initial length $l_{0b}$ (Fig. 6) extended to length $l_{1b}$. The ratio of the two lengths gives

$$\frac{l_{1b}}{l_{0b}} = 1 + a_b \tag{40}$$

where $a_b$ is the bifilament strain. But

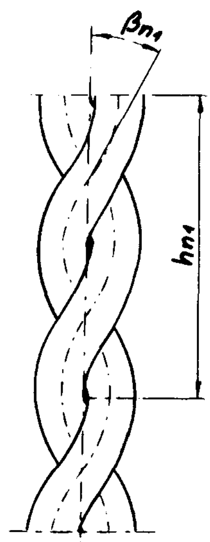



Fig. 6. Schematic representation of a twisted and stretched bifilament.

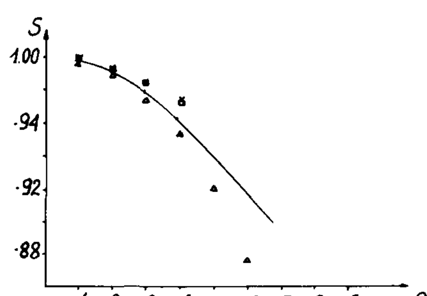



Fig. 7. Experimental coefficient of retraction as a function of twist parameter: (●) $d$ = 0.80 mm; (×) $d$ = 0.61 mm; (□) $d$ = 0.50 mm; (■) $d$ = 0.21 mm.

$$l_{0b} = l_{0m} \cos \beta_{b0} \tag{41}$$

where $l_{0m}$ is the monofilament length in unstrained bifilament and $\beta_{b0}$ is the angle between a tangent to the monofilament axis and the direction of the bifilament axis prior to strain. Also,

$$l_{1b} = l_{1m} \cos \beta_{b1} = l_{0m} (1 + \epsilon_m) \cos \beta_{b1} \tag{42}$$

where $\epsilon_m$ is the monofilament strain and $\beta_{b1}$ is the angle between a tangent to monofilament axis and bifilament axis, in a strained bifilament.

The geometric relations give

$$\cos \beta_{b0} = \frac{1}{\sqrt{1 + g_b{}^2}} \tag{43}$$

and

$$\cos \beta_{b1} = \frac{1}{\sqrt{1 + \mathrm{tg}^2 \beta_{b1}}} = \frac{1}{\sqrt{1 + \left(2\pi\, uR \dfrac{T_b}{1 + a_b}\right)^2}} = \frac{1}{\sqrt{1 + g_b^2 \dfrac{1}{(1 + a_b)^2 (1 + \epsilon_m)}}} \tag{44}$$

if it is assumed that the square coefficient of monofilament constriction $u^2$ is approximately inversely proportional to the monofilament length after strain $(1 + \epsilon_m)$.

If the values are substituted in eq. (40), the following equation is obtained:

$$1 + a_b = \frac{l_{0m} (1 + \epsilon_m) \cos \beta_{b1}}{l_{0m} \cos \beta_{b0}} = (1 + \epsilon_m) \times \frac{1 + a_b}{\sqrt{(1 + a_b)^2 + \dfrac{g_b^2}{1 + \epsilon_m}}} \sqrt{1 + g_b^2} \tag{45}$$

Upon simple transformation, an equation is obtained which interrelates the monofilament and bifilament strains in the form

$$(1 + \epsilon_m)^3 - \frac{(1 + a_b)^2}{1 + g_b^2} (1 + \epsilon_m) - \frac{g_b^2}{1 + g_b^2} = 0 \tag{46}$$

from which the monofilament strain $\epsilon_m$ can be calculated for adopted values of the bifilament twist parameter $g_b$ and strain $a_b$.

Following a previous determination of the relationship between the monofilament residual strain and total strain $\epsilon_{mp}(\epsilon_{mt})$, it is possible to establish the bifilament residual strain $a_{bp}$ from the equation

$$(1 + a_{bp})^2 = (1 + \epsilon_{mp})^2 (1 + g_b^2) - \frac{g_b^2}{1 + \epsilon_{mp}} \tag{47}$$

## EXPERIMENTAL

### Investigating the Filament Retraction Factor

For the purposes of determing the filament retraction factor, a special testing station had been arranged. The station consisted of a twist tester head, of which the spindle was located so that its axis was positioned vertically, and a movable second clamp which was located under the spindle. The distance between the spindle clamp and the movable clamp was 600 mm.

In the clamps, twistless filament lengths were fixed and preloaded to 5 mN·m/mg. Then, the filament was twisted and the shortening of the filament length $\lambda$ as a function of the number of the spindle revolutions was recorded. The retraction factor was calculated from the formula

$$s = \frac{l_0 - \lambda}{l_0} \tag{48}$$

and the twist parameter from the formula

$$g = \pi d n \frac{1}{l_0 - \lambda} \tag{49}$$

where $l_0$ = initial length of specimen (distance between the clamps), $d$ = initial diameter of specimen, and $n$ = number of spindle revolutions.

Analyzed were polyester filaments of diameter $d$ = 0.21 mm, 0.50 mm (dull), 0.61 mm, and 0.80 mm (dull). In all cases, an almost complete coincidence of the calculated and experimental values of retraction factor for a twist parameter interval of $0 \leqslant g \leqslant 0.3$ was observed (Fig. 6). Differences were observed with higher values of the twist parameter; namely, for the retraction factor, the experimental value was lower than the calculated. The differences may be the result of (1) different values of Young's modulus, in the stretching and compression zone, at strains greater than 0.02 (2%); or (2) differentiation of Young's modulus between the filament external layer and center, which has been postulated as an explanation of certain peculiarities in the relationship between shear modulus and filament diameter.[2] The second hypothesis seems more plausible, since in the case of finer filaments the outer layer takes relatively more of the filament cross-sectional area than in a coarse filament.

However, a full explanation of the differences would require a more comprehensive study based on a much wider range of experimental material.

### Change of Filament Diameter in Stretching

In the heretofore discussion the change of filament diameter in stretching was included. Frequently, it is assumed *a priori* that, in stretching, the filament volume remains constant and therefore

$$\frac{\pi d_0^2}{4} l_0 = \frac{\pi d_1^2}{4} l_1 \tag{50}$$

where $d_0$, $d_1$ is the filament diameter, respectively, in the initial state and after stretching; and $l_0$, $l_1$ is the filament length, respectively, in the initial state and after stretching. From this equation the coefficient of constriction is calculated:

$$u = \frac{d_1}{d_0} = \sqrt{\frac{l_0}{l_1}} = (1 + a)^{-0.5} \tag{51}$$

In order to verify the above postulate, an investigation of the filament diameter change in stretching was carried out. The testing equipment consisted of a biological microscope provided with a micrometric eyepiece and stretching device slidably mounted on the microscope stage. Filament diameter was measured in places marked on the test specimen in its initial state and after the specimen had been stretched by predetermined values. The reading accuracy was 0.001 mm. The results are presented in Figure 8.

From analysis of the results it can be seen that the diameter change versus strain of the specimen can be described by this approximated relation:

$$u = \frac{d_1}{d_0} = (1 + a)^{-y} \tag{52}$$

where $y = 0.42$–$0.50$ is dependent on the type of the monofilament. If $y < 0.50$, the volume of the monofilament, as can be easily seen, increases.

## Measurement of Filament Strength and Elongation

Serious difficulties were encountered when measuring the strength and elongation of filaments, particularly of twisted filaments. All tests were performed on the Instron Universal tensile tester, at first using the flat pneumatic clamps. With these clamps, however, a considerable number of the twisted filaments were broken near one of the clamps. This meant that there was a jump of stress near the clamp, which weakened the specimen. On the other hand, some twistless filaments would partly creep out of the clamps, which distorted the picture of the load–elongation relation.

It was, therefore, decided to carry out the tests using the clamps for testing cords. These, however, did not provide a precise picture of the relationship between the stretching load and elongation, nor was it possible to obtain a precise value of the elongation at break. In order to obviate the difficulty, it was decided to use twin specimens: one with interclamp distance $l'$ (50 mm) and the other with interclamp distance $l''$ (200 mm). This enabled one to obtain a net specimen length $l'' - l' = 150$ mm, and preliminary tests showed that the stretching time till break was identical for both specimens.

Upon averaging the load–elongation curves for each series of specimens, it was possible, by subtracting the shorter specimen elongation $\lambda'$ from the longer specimen elongation $\lambda''$ at the same load, to obtain the elongation at this load for the 150-mm-long specimen. The results obtained for four filaments are presented in Tables I and II.

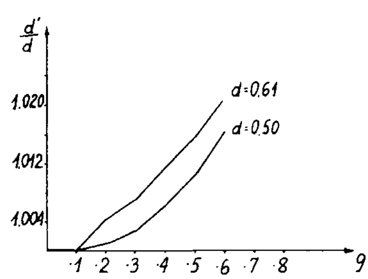



Fig. 8. Changes of filament diameter vs. strain.

TABLE I

Breaking Load of Samples at the Initial Distance between Hooks of 50 mm ($F$) and 200 mm ($F$)[a]

| Twist param. $g$ | Breaking Load 0.21 mm[b] $F$ [N] | $F$ [N] | 0.50 mm $F$ [N] | $F$ [N] | 0.61 mm $F$ [N] | $F$ [N] | 0.80 mm $F$ [N] | $F$ [N] |
|---|---|---|---|---|---|---|---|---|
| 0 | 21.5 | 21.6 | 108.9 | 109.1 | 142.9 | 141.7 | 249.4 | 249.2 |
| | (0.4) | (0.4) | (1.2) | (1.3) | (1.3) | (1.2) | (6.0) | (8.3) |
| 0.10 | 21.2 | 20.5 | 110.5 | 109.6 | 139.6 | 138.7 | 251.0 | 248.9 |
| | (0.3) | (0.3) | (1.2) | (1.7) | (2.2) | (2.5) | (8.2) | (6.8) |
| 0.20 | 21.4 | 20.5 | 107.7 | 105.2 | 140.5 | 140.2 | 246.8 | 244.7 |
| | (0.3) | (0.2) | (1.2) | (3.8) | (1.2) | (1.9) | (7.5) | (6.5) |
| 0.31 | 21.0 | 20.2 | 107.1 | 104.4 | 140.2 | 138.9 | 246.8 | 242.6 |
| | (0.5) | (0.4) | (1.5) | (3.4) | (1.9) | (1.7) | (8.6) | (5.4) |
| 0.41 | 20.7 | 19.8 | 105.8 | 102.2 | 142.9 | 139.8 | 244.5 | 240.9 |
| | (0.7) | (0.7) | (2.2) | (4.0) | (3.2) | (1.8) | (6.6) | (6.4) |

[a] Figures in parentheses are standard deviations of breaking load.

[b] Filament diameter.

Analysis of the results shows that the breaking load is only slightly reduced with increase in twist parameter. The elongation of samples 50 and 200 mm long increases with increasing twist. However, the differences between these two values for every kind of filament and twist parameter do not coincide with elongations of broken samples.

Of the adopted method, which is based on difference of elongations, it can be said that while it is adequate in describing the load–elongation relationships, it fails to give fully reliable results where critical values are involved. The reasons are as follows.

Obviously, the longer specimens have a higher internal fineness irregularity, and this means lower mean breaking load and lower mean elongation at break.[3] The load–elongation curve is almost identical for specimens of different lengths. If, however, creeping of the specimen occurs in the critical zone, then even with increased specimen length (and, therefore, increased fineness irregularity) the

TABLE II

Elongation of Samples at Initial Distance between Hooks of 50 mm ($\lambda'$) and 200 mm ($\lambda''$), and Calculated Elongation of Samples at 150 mm ($\lambda'' - \lambda'$)[a]

| Twist param. $g$ | Elongation 0.21 mm[b] $\lambda'$ | $\lambda''$ | $\lambda'' - \lambda'$ | 0.50 mm $\lambda'$ | $\lambda''$ | $\lambda'' - \lambda'$ | 0.61 mm $\lambda'$ | $\lambda''$ | $\lambda'' - \lambda'$ | 0.80 mm) $\lambda'$ | $\lambda''$ | $\lambda'' - \lambda'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 24.9 | 48.9 | 24.0 | 54.4 | 109.1 | 54.7 | 51.4 | 100.6 | 49.2 | 36.2 | 68.4 | 32.2 |
| | (3.1) | (2.9) | | (0.2) | (0.2) | | (3.0) | (3.8) | | (3.8) | (4.7) | |
| 0.10 | 27.3 | 51.8 | 24.5 | 55.1 | 106.4 | 51.3 | 51.7 | 99.6 | 47.9 | 37.1 | 69.6 | 32.5 |
| | | (4.0) | | (0.2) | (0.2) | | (3.4) | (3.8) | | (4.9) | (4.1) | |
| 0.20 | 28.5 | 52.5 | 24.0 | 55.7 | 102.5 | 46.8 | 51.0 | 97.6 | 46.6 | 38.5 | 72.6 | 34.1 |
| | | (3.9) | | (4.3) | (10.0) | | (3.5) | (4.8) | | (4.5) | (5.3) | |
| 0.31 | 28.5 | 52.6 | 24.1 | 56.7 | 103.5 | 46.8 | 49.9 | 96.7 | 46.8 | 38.9 | 73.2 | 34.3 |
| | | (3.9) | | (3.9) | (9.9) | | (3.4) | (4.4) | | (4.2) | (5.1) | |
| 0.41 | 29.8 | 52.2 | 22.4 | 55.7 | 100.3 | 44.6 | 50.8 | 100.0 | 49.2 | 39.6 | 76.8 | 37.2 |
| | | (3.9) | | (0.2) | (0.5) | | (3.7) | (4.8) | | (4.5) | (5.3) | |

[a] Figures in parentheses are standard deviations of elongation.

[b] Filament diameter.

creeping will not show in the diagram, since it affects only a small part of the specimen. In subtracting $\lambda'$ from $\lambda''$, the obtained elongation is less then the actual elongation. This was particularly strongly manifested in the case of the filament of $d = 0.21$.

It must be mentioned that in certain cases visible creeping was followed by decrease in load at increasing elongation.

To show better the changes in the strength of filaments with increasing twist, the relationship of the strength of filament with the $g$ twist parameter, $\sigma_f(g)$, to the strength of untwisted parameter, $\sigma_f$ (0), was calculated as follows:

For experimental results:

$$C = \frac{\sigma_f(g)_{\text{exp}}}{\sigma_f(0)_{\text{exp}}} \tag{53}$$

For calculated values of $\sigma_f(g)$ from eq. (31) and $\sigma_f(0)$ from eq. (30):

$$C' = \frac{\sigma_f(g)}{\sigma_f(0)} \tag{54}$$

For calculated values of $\sigma_f(g)$ and $\sigma_f(0)$ from the same equations assuming creep of material at breaking elongation, i.e., $H = 0, J = 0$:

$$C'' = \frac{\sigma_f(g)}{G} = \frac{2k}{g^2} \ln \frac{\sqrt{1+g^2} + \sqrt{k^2+g^2}}{1+k} \tag{55}$$

The results are presented in the Figures 9(a) through 9(d).

It can be seen that there is better coincidence of values of $C''$ with values of $C$. However, the comparison of the stresses calculated for strains smaller than critical shows satisfactory agreement of experimental with values computed from eq. (31).

## Elasticity of Bifilaments

The experimental material consisted of two polyamide monofilaments of diameter $d = 0.15$ and 0.25 mm, respectively. The mechanical properties of both the monofilaments and the bifilaments were tested on the Instron tensile tester.

The specimens of twistless and twisted monofilaments and bifilaments were fixed in flat clamps; since the specimens ruptured near the clamps, the ranges of load and total elongation were limited. But in tests of elasticity, this did not have any significant effect on the results.

The specimens, both monofilaments and bifilaments, were tested with the initial interclamp distance being 200 mm. Preloading was used to 5 mN·m/mg. Thus clamped, each specimen was successively elongated by 4, 8, 12, 16, and 20 mm in relation to its initial length. Both at each highest point and each time the lower clamp returned to its starting position, the lower beam was stopped at 60 sec. The tests were performed at $t$ = 20–24°C and at a humidity of 60–65%.

The monofilaments were tested after they had been twisted to the following twist parameter values:

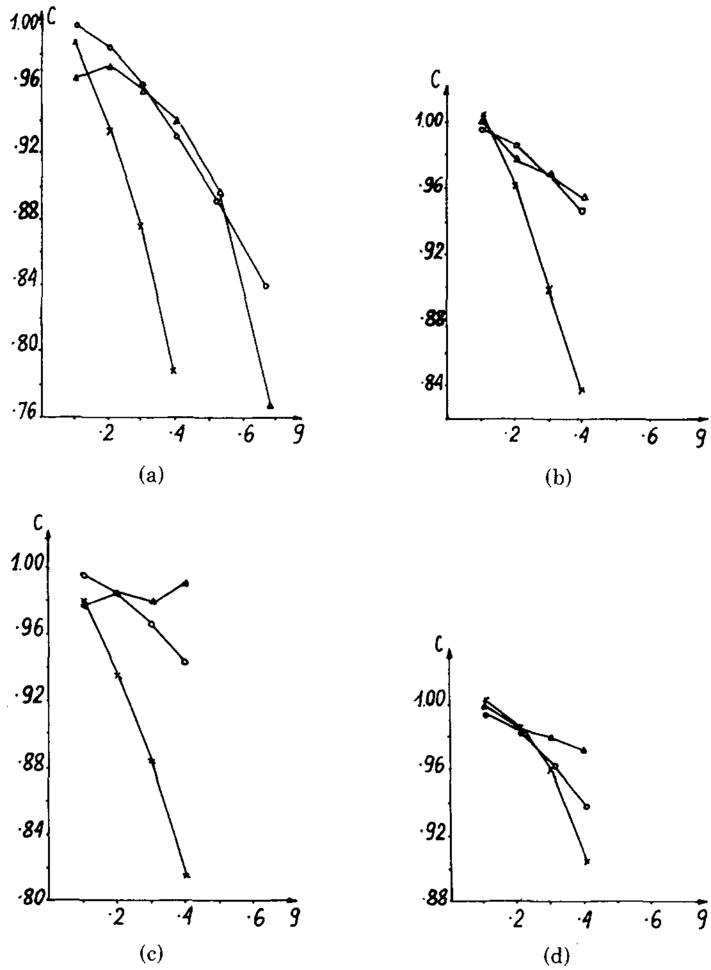
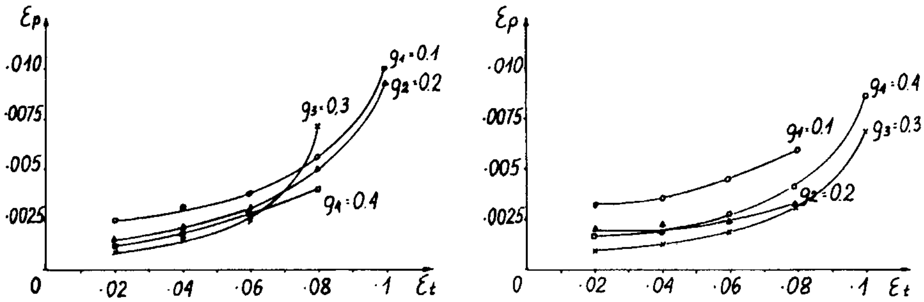



Fig. 9. Filament strength, experimental and calculated from eqs. (30) and (31), as function of twist parameter: (a) $d$ = 0.21 mm; (b) $d$ = 0.50 mm; (c) $d$ = 0.61 mm; (d) $d$ = 0.80 mm; (×) $C'$; (○) $C''$; (△) experimental.

| Monofilament diameter | $d$ = 0.15 mm | $d$ = 0.25 mm |
|---|---|---|
| $g^{i}$ | 0.092 | 0.108 |
| $g^{ii}$ | 0.196 | 0.199 |
| $g^{iii}$ | 0.269 | 0.299 |
| $g^{iv}$ | 0.392 | 0.392 |

The twisting was effected on a special twist tester. The values of residual strain $a_p$ vs. total strain $a_t$ are presented in Figures 10(a) and 10(b). Then, using the same twist tester, the bifilaments were twisted so that the twist parameter was $g_n$ = 0.2, $g_n$ = 0.4. The values of residual strain $a_p$ vs. total strain $a_t$ are presented in Figures 11(a) and 11(b).

Using the twist parameter values for the mono- and bifilaments and values of residual strain versus total strain for the monofilaments, the bifilament residual strains at predetermined total strains were calculated from eq. (47). The




(a) (b)

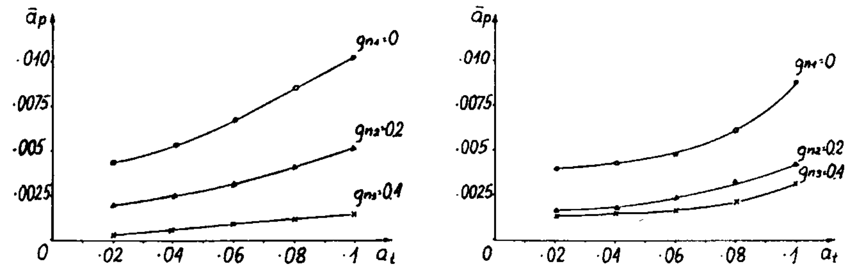
Fig. 10. Residual strain vs. total strain in monofilaments: (a) $d$ = 0.15 mm; (b) $d$ = 0.25 mm.

calculated and experimental results are listed in Table III. As can be seen in the table there is a high agreement between the calculated and the experimental results.

## CONCLUSIONS

The results obtained in testing the mechanical properties of the discussed filaments and bifilaments and compared with results of mathematical analysis permit to formulate the following conclusions:

(1) It is possible to predetermine analytically the filament contraction in twisting with a twist parameter interval $0 \leqslant g \leqslant 0.4$. The differences that are observed with higher levels of twist require further studies.




(a) (b)

Fig. 11. Residual strain vs. total strain in bifilaments: (a) $d$ = 0.15 mm; (b) $d$ = 0.25 mm.

TABLE III
Calculated from Equation (47) and Experimental Bifilament Residual Strain as a Function of Total Strain[a]

| Bifilament total strain $a_{bt}$ | Bifilament residual strain $a_{bp}$, % | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $d$ = 0.15 mm[a] | | | | $d$ = 0.25 mm | | | |
| | $g_n$ = 0.196 | | $g_n$ = 0.392 | | $g_n$ = 0.199 | | $g_n$ = 0.398 | |
| | Cal'd | Exper. | Cal'd | Exper. | Cal'd | Exper. | Cal'd | Exper. |
| 2 | 0.15 | 0.19 | 0.17 | 0.14 | 0.16 | 0.17 | 0.17 | 0.18 |
| 4 | 0.26 | 0.25 | 0.23 | 0.20 | 0.20 | 0.18 | 0.23 | 0.21 |
| 6 | 0.31 | 0.32 | 0.29 | 0.27 | 0.24 | 0.26 | 0.29 | 0.24 |
| 8 | 0.46 | 0.42 | 0.52 | 0.43 | 0.36 | 0.35 | 0.46 | 0.37 |
| 10 | 0.98 | 0.53 | 1.09 | 0.97 | 0.67 | 0.44 | 0.98 | 0.86 |

[a] Filament diameter

(2) Analysis of the changes in the diameter of polyester filaments, which are observed in tensile testing, suggests that the reasons of the varied behavior of the filaments are to be sought in the initial processing.

(3) The strength of twisted filaments decreases with increasing twist, and comparison of the experimental and the calculated results implies that a decisive part is played here by material creeping in the final stage of stretching.

(4) In twisted filaments, the elongation at break increases with higher twist.

(5) In bifilaments, the residual strain versus total strain can be predicted on the basis of the monofilament properties and bifilament twist.

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